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Regular paper

Blind MMA Equalization for QAM modulation in 5G/6G communications

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The rapid evolution of wireless communication systems, particularly the development of 5G and the emerging 6G networks, has ushered in a new era of connectivity characterized by ultra-high data rates, low latency, and reliable communication.These advancements necessitateadvanced modulation techniques, such as Quadrature Amplitude Modulation (QAM), to achieve the spectral efficiency required for modern applications. However, high-order QAM constellations, while enabling efficient bandwidth utilization, are inherently more vulnerable to noise, interference, and channel-induced distortions, leading to inter-symbol interference (ISI) and increased bit error rates (BER). Effective equalization techniques are therefore indispensable to mitigate these effects and maintain robust communication performance. Blind equalization has emerged as a vital solution to address these challenges, as it eliminates the need for training sequences, thereby improving bandwidth efficiency. Among the various blind equalization methods, the Multi-Modulus Algorithm (MMA) has proven particularly effective for high-order QAM modulation. In this paper, we introduce an enhanced MMA algorithm designed to address these limitations and optimize performance for 5G and 6G communication systems. The proposed algorithm incorporates several key innovations, such as Dynamic Step Size Adaptationand Advanced Weight Initialization. The effectiveness of the proposed algorithm is demonstrated through extensive MATLAB simulations for high-order QAM schemes under challenging multipath fading and noise conditions. Performance metrics such as BER are compared with those of conventional MMA implementations and other state-of-the-art blind equalization techniques. The results indicate that the proposed algorithm significantly outperforms traditional methods, achieving faster convergence, lower BER, and greater robustness to noise and interference.

Keywords:MMA; 5G/6G; M-QAM; Blind Equalization.

1. Introduction

The rapid advancements in wireless communication technologies have revolutionized how information is transmitted and received, enabling a connected world with unprecedented speed, reliability, and efficiency [1]. From the advent of 5G networks to the exploration of 6G and beyond, there is a growing demand for high data rates, low latency, and seamless connectivity [2]. These next-generation communication systems aim to support diverse applications such as autonomous vehicles, virtual reality, smart cities, and ultra-reliable low-latency communication (URLLC), all of which place immense pressure on network infrastructure and modulation techniques [3].

Among the various modulation schemes,Quadrature Amplitude Modulation (QAM) stands out as a cornerstone in achieving the high spectral efficiency needed for 5G and 6G

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networks [4]. By combining amplitude and phase modulation, QAM maximizes the use of available bandwidth, transmitting multiple bits per symbol. Higher-order QAM, such as 64-QAM and 256-QAM, is particularly beneficial for meeting the data-intensive requirements of modern applications [5]. However, these benefits come at the cost of increased sensitivity to noise, interference, and channel distortions, posing significant challenges to reliable data transmission.

In this context, signal equalization becomes a critical component of wireless communication systems [6]. Equalization mitigates the detrimental effects of inter-symbol interference (ISI) and channel-induced distortions that can degrade signal quality [7]. Traditional equalization methods often rely on training sequences, which consume valuable bandwidth and reduce system efficiency [8]. This limitation has driven the development of Blind Equalization Techniques, which do not require prior knowledge of the transmitted signal [9]. By estimating and correcting distortions based solely on the received signal, blind equalization techniques enhance system efficiency and robustness [10].

Among the blind equalization algorithms, the Multi-Modulus Algorithm (MMA) has gained significant attention due to its effectiveness in handling QAM signals [11]. MMA builds upon the Constant Modulus Algorithm (CMA) by introducing separate cost functions for the in-phase and quadrature components of the signal. This approach allows MMA to achieve superior performance in reducing ISI and aligning received symbols with their ideal constellation points [12]. MMA's ability to work seamlessly with high-order QAM constellations makes it a preferred choice for modern communication systems [13].

In this paper, we introduce an enhanced MMA method designed specifically to address the demanding requirements of 5G and 6G communication systems. The proposed algorithm incorporates several key improvements, including dynamic step size adaptation, tailored initialization for equalizer weights, modulation-specific optimization, and precise delay evaluation. These enhancements provide faster convergence, improved stability, and better performance under varying noise and channel conditions compared to traditional MMA implementations.

The proposed approach not only mitigates ISI effectively but also ensures robust performance across higher-order QAM constellations, thereby addressing critical challenges in wireless communication. The findings underscore the potential of the advanced MMA algorithm to significantly enhance system performance, paving the way for its integration into next-generation communication technologies.

The remainder of the manuscript is organized as follows. The literature review is presented in Section 2. The system model is described in Section 3. The simulation findings and discussions are covered in Section 4, and the final conclusion is presented in Section 5.

2. The literature review

Here we'll have a look at the work done in the field of blind equalization to eliminate ISI. Paper [14] discussed semi-blind and blind equalizers for fading MIMO systems and proposed channel-specific methods. It was found that semi-blind and blind algorithms outperform training-based equalizers in Ricean environments, whereas training-based equalizers are more efficient in Rayleigh channels. The article also identified the optimal training size for semi-blind and training algorithms. In paper [15], H. Bellahsene and al, use the calculation of maximum kurtosis to determine the optimal time domain equalizer (TEQ) and synchronization delay for the maximum signal-to-noise ratio shortening (MSSNR) algorithm and compare the optimal filters obtained by other methods and the proposed method based on maximum kurtosis. Paper [16] compared Sato's Algorithm and Godardbased blind algorithms for PAM signals and found that using variable values for the tap adjusting coefficient and step size based on iterations could speed up convergence and eliminate maladjustments. Numerous studies have proposed the use of CMA or MMA algorithms. In a paper [17], J. van der Veen and al, introduced a combination of the MRE algorithm with the constant modulus algorithm (CMA) for single-input multiple output (SIMO) systems, enhancing its performance in blind equalization. Paper [18] developed a blind equalization method that outperforms supervised algorithms without regard to QAM order, providing faster convergence and stability gains over decision-directed algorithms. In the paper [19], the authors compared the performance of CMA, stop-and-go decision directed, and Wei Rao's CMA for QAM schemes in a linear band-limited channel. They concluded that the combined channel and equalizer impulse response, after convergence, corresponds to the ideal channel impulse response. The stop-and-go method outperformed other algorithms in terms of MSE and convergence rate.Paper [20] discussed the Multi Modulus Algorithm (MMA), a modified variation of CMA that does not require a separate phase recovery system. This new model demonstrated improved data speeds and lower BER. Paper [21] described a method for blind multi-modulus equalization that calculates expenses using the Constant Modulus Algorithm and switches to MMA at specified thresholds. This iterative procedure is repeated until a preset value is reached, and the equalization coefficients are changed accordingly. Paper [22] reviews four equalization algorithms, two training-based and two blind, for 16-QAM and 64-QAM modulation schemes to address Inter-Symbol Interference (ISI) in wireless communication systems. The study compares the performance of these algorithms in terms of bit error rate (BER), residual ISI, and mean square error (MSE) through simulations. Results show that the Recursive Least Squares (RLS) algorithm slightly outperforms the Least Mean Square (LMS) algorithm at higher signal-to-noise ratios. The paper concludes by suggesting future comparisons of these algorithms to determine the best fit for emerging 5G and 6G communication technologies.Paper [23] introduces the VSS-FC-MMA algorithm, an enhancement of the FC-MMA adaptive equalization method using a varying step size to improve equalization performance and reduce inter-symbol interference (ISI) in non-constant modulus signal transmission. While FC-MMA enhances convergence speed, it suffers in steady-state performance due to its fixed step size. The proposed VSS-FC-MMA improves steady-state performance by adjusting the step size dynamically. Simulation results show that VSS-FC-MMA outperforms FC-MMA in terms of residual ISI, mean square error (MSE), and symbol error rate (SER), though it has a 1.7x slower convergence time. Paper [24] introduces the Manifold Optimization Aid Modified Multi-Modulus Algorithm (MO-M3A) for improving MIMO blind detection, which aims to reduce training overhead while addressing performance issues and slow processing speeds. The MO-M3A leverages a novel weighted multi-modulus loss function to mitigate interference in the traditional MMA loss function, enhancing signal recovery. By restricting the solution to the Stiefel manifold and applying the Riemannian gradient method, the algorithm simplifies the problem and accelerates iterations. Simulation results show that MO-M3A achieves better bit error rate (BER) performance and reduces processing burden compared to conventional methods.

3. System model

3.1.Quadrature Amplitude Modulation (QAM)

QAMis a foundational technique in modern digital communication systems that combines amplitude and phase modulation to maximize data transmission efficiency. By leveraging both in-phase (I) and quadrature (Q) components, QAM enables the transmission of multiple bits per symbol, thus achieving high spectral efficiency [25].

The M-QAM variant, where "M" represents the number of unique symbols in the constellation, further enhances efficiency by encoding multiple bits per symbol. As M increases, more bits can be transmitted per symbol [26]. For example, in 16-QAM, each symbol encodes 4 bits, while in 64-QAM and 256-QAM, each symbol represents 6 and 8 bits, respectively, leading to significantly higher data rates.

In M-QAM, the modulated signal is expressed as a linear combination of two orthogonal carrier waves, sinusoidal functions modulated by the I and Q components. The components correspond to the real and imaginary parts of a complex signal, which are represented as distinct points on a constellation diagram. The mathematical representation of a symbol is given by:

$$S_{i,j} = I_i + j\,Q_j \qquad (1)$$

Where:

- I_i and Q_j are the amplitude levels for the in-phase and quadrature components, respectively.
- j is the imaginary unit.

The time-domain representation of the modulated signal can be expressed as:

$$S(t) = I(t)\cos(2\pi f_c\, t) - Q(t)\, sin\,(2\pi f_c\, t) \qquad (2)$$

Where:

- $f_c$is the carrier frequency.
- $I(t)$and $Q(t)$are the baseband signal components representing the data to be transmitted.

The number of bits per symbol is defined as $log_2 M$, allowing M-QAM to support increasingly dense constellations as M grows.

While higher-order M-QAM schemes (e.g., 256-QAM) enable increased data rates by utilizing denser constellations, they also introduce greater vulnerability to noise and interference. This creates a critical trade-off between spectral efficiency and bit error rate (BER) performance. The susceptibility to noise increases as the distance between constellation points decreases, making error correction and channel equalization essential in maintaining communication reliability.

These trade-offs are particularly relevant in advanced applications such as:

- 5G Networks: Utilizing higher-order QAM, such as 64-QAM, to meet the stringent requirements of high data rates, low latency, and efficient bandwidth utilization [27].

- 6G Systems: Expected to incorporate ultra-high-order QAM (e.g., 256-QAM) for terahertz (THz) communications, supporting data-intensive applications [28].

3.2. The Multi-Modulus Algorithm (MMA)

The Multi-Modulus Algorithm (MMA) is an extension of the Constant Modulus Algorithm (CMA), specifically designed to handle quadrature amplitude modulation (QAM) and quadrature phase-shift keying (QPSK) signals [29]. MMA operates by independently equalizing the in-phase (I) and quadrature (Q) components of the signal, which enables it to address inter-symbol interference (ISI) and channel distortions more effectively. This separation facilitates minimizing symbol dispersion around their ideal constellation points in each signal dimension [30].

MMA achieves this by minimizing two distinct cost functions, one corresponding to the I-component and the other to the Q-component of the signal. The cost function is expressed as follows:

$$J_{MMA}(w) = E\,[\left(|I(n)|^2 - R_I\right)^2 + (|Q(n)|^2 - R_Q)^2] \tag{3}$$

- $I(n)$ and $Q(n)$ are the in-phase and quadrature components of the output signal $y(n)$ at a time n.
- R_I and R_Q are constants representing the target moduli for the in-phase and quadrature components, determined by the specific signal constellation.
- $w$represents the vector of equalizer coefficients.
- $E[\cdot]$is the expectation operator.

To minimize this cost function, the MMA algorithm uses a gradient descent method to iteratively adjust the equalizer coefficients. The coefficient update rule is given by:

$$w_{n+1} = w_n - \mu \,.\, \nabla J_{MMA}(w_n) \tag{4}$$

- $\mu$is the step size (controlling the convergence speed of the algorithm).
- $\nabla J_{MMA}(w_n)$is the gradient of the cost function concerningthe filter coefficients w_n .

This iterative adjustment aims to reduce distortion and align the recovered symbols closely with their respective constellation points.

MMA has proven to be a robust technique for blind equalization in mobile communication systems, particularly forQAM modulation schemes. These modulation methods are extensively employed in 5G networks due to their high data rate capabilities. In the context of massive MIMO systems in 5G, MMA effectively mitigates channel distortions, ensuring reliable and low-latency communication [31].

Looking ahead to 6G networks, the algorithm is expected to play a pivotal role in enabling terahertz (THz) communications and supporting emerging applications requiring ultra-high data rates and minimal latency. This includes immersive technologies, such as holographic communications and extended reality (XR), further emphasizing MMA's relevance in advancing next-generation communication systems [32].

In this study, we propose an advanced MMA method incorporating several key enhancements to improve blind equalization and optimize communication system performance. The primary distinctions between our approach and the standard MMA method are as follows:

1. Adaptive Step Size: Unlike the standard MMA, which uses a fixed step size (μ) throughout the algorithm.Our method dynamically adjusts the step size based on theinstantaneous errorΦ_n using a linear decay formula:

$$\mu = \mu \cdot (1 - 0.1 \cdot |\Phi_n|) \tag{5}$$

   This adaptation accelerates convergence and minimizes oscillations or instability, especially for higher modulation orders such as 256-QAM.

2. Equalizer Weight Initialization: The weights of the equalizer are initialized with zeros, except for the 21st weight, which is set to 1. The 21st weight set to 1 introduces a targeted offset that helps the algorithm establish a baseline for equalization, particularly useful in dispersive or complex channel environments. Standard MMA methods often rely on random or constant initialization without this specificity.

3. Modulation-Specific Optimization: Our method employs different step sizes for each QAM modulation order (16, 64, 256). This tailored approach enables precise control of stability and performance for varying modulation levels, a feature not typically available in standard MMA implementations.

4. Lower Bound for Step Size:The step size is constrained to a minimum value 1×10^{-12}, preventing it from becoming excessively small and slowing down the equalization process. This safeguard is not part of the standard MMA method, where the step size remains fixed.

5. Delay Evaluation:Our implementation evaluates the delay between the equalized signal and the original signal using cross-correlation. This delay is accounted for in the bit error rate (BER) calculation, ensuring accuracy. Standard MMA implementations often overlook this delay, leading to less precise BER evaluations.

The proposed program includes specific enhancements that make the MMA more adaptable and optimized for high-order modulation. These improvements, such as dynamic step size adjustment, targeted weight initialization, modulation-specific optimizations, and precise BER calculations, contribute to faster convergence and improved system performance in noisy channel conditions. These advancements position our method as a robust solution for modern communication systems, particularly in the context of 5G and future 6G networks, where high data rates and reliable performance are critical.

4. Simulation Results and discussions

In this section, we present the MATLAB simulation results obtained using the advanced MMA blind equalization method for M-QAM modulation schemes. The performance metric evaluated is the Bit Error Rate (BER), which measures the ratio of incorrectly received bits to the total number of transmitted bits.

The obtained results are then compared with the findings reported in the literature, particularly with the results in the paper by Ahmed et al. (2019) [22], which reviews various training and blind equalization algorithms for wireless communications.

The simulation was conducted using a M-QAM (16-QAM, 64-QAM, and 256-QAM)modulation scheme. A total of 10^6 symbols were transmitted over a multi-path fading channel characterized by an impulse response of h=[0.2,0.9,0.3], simulating typical 5G channel conditions. The signal-to-noise ratio (Eb/N0) was varied from -10 to 40 dB to observe performance across different noise levels. Additive White Gaussian Noise (AWGN) was added to the transmitted signal, with noise variance adjusted according to the specific Eb/N0 values.

Figures 1,2, and 3, depict the BER performance of the MMA algorithm for QPSK, 16-QAM, 64-QAM, and 256-QAM modulation schemes, respectively.

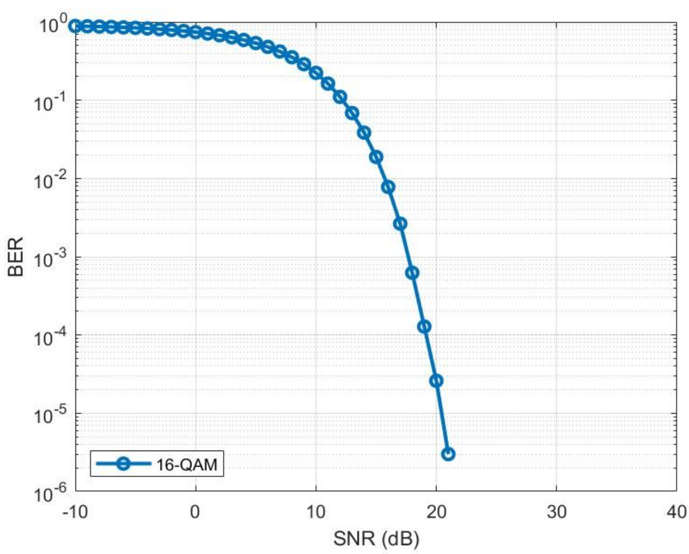

Figure 1: BER of MMA algorithm for a 16-QAM constellation.

For 16-QAM (Figure 1), the algorithm achieves a rapid BER decrease, reaching values below 10^{-6} at an SNR of 22 dB, showcasing excellent noise resilience for lower-order modulation schemes.

For 64-QAM (Figure 2), the MMA algorithm continues to perform robustly, with the BER dropping below 10^{-6}at an SNR of 28 dB. However, the BER curve shifts slightly to the right compared to 16-QAM, reflecting the higher complexity and susceptibility to errors associated with 64-QAM.

Similarly, for 256-QAM (Figure 3), the MMA algorithm maintains strong performance, achieving a BER below 10^{-6} at an SNR of 38dB. The further rightward shift of the curve highlights the additional challenges posed by the higher complexity and error rates inherent in 256-QAM. These results underscore the advanced MMA algorithm's adaptability and effectiveness across varying modulation orders in mitigating BER under different SNR conditions.

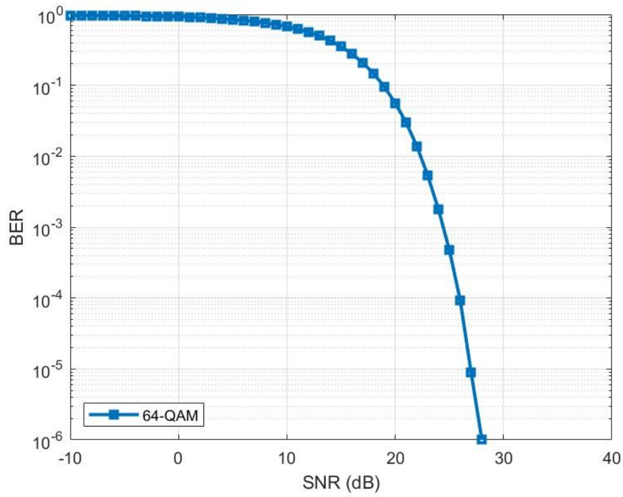

Figure 2: BER of MMA algorithm for a 64-QAM constellation.

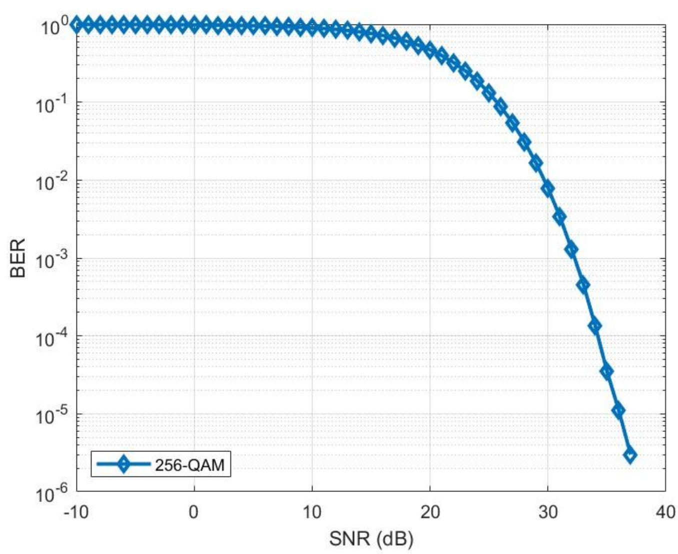

Figure 3: BER of MMA algorithm for a 256-QAM constellation.

Figures 4, 5, 6,and 7, are the results of the paper by Ahmed et al. (2019) [22], which reviews various training and blind equalization algorithms for wireless communications.

Figure 4 illustrates the BER performance of the MMA and SCA algorithms for a 16-QAM modulation scheme over varying Signal-to-Noise Ratios (SNR). By the time the SNR reaches 20 dB, the MMA-16QAM achieves a BER close to 10^{-4}, and the SCA-16QAM achieves a BER equal a 10^{-2}.

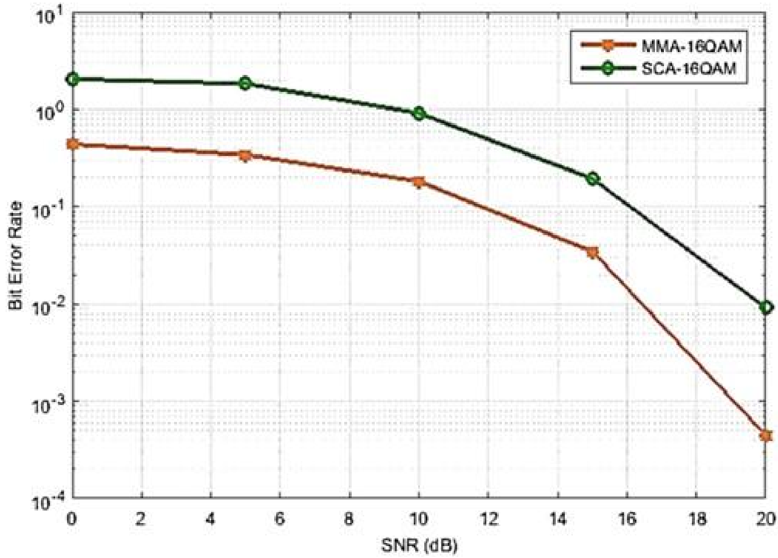

Figure 4: BER comparison of MMA and SCA algorithm for 16-QAM constellation [22].

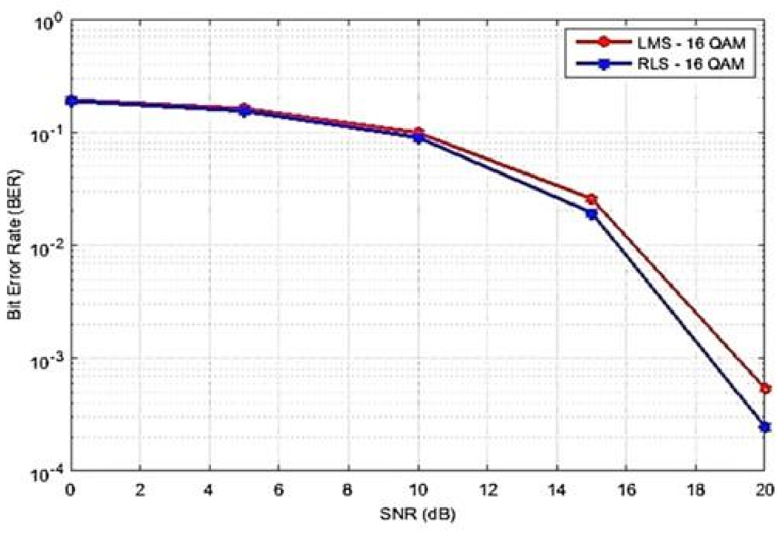

Figure 5: BER comparison of LMS and RLS algorithm for a 16-QAM constellation[22].

Figure 5 illustrates the BER performance of the LMS and RLS algorithms for a 16-QAM modulation scheme over varyingSNR. By the time the SNR reaches 20 dB, the LMS and RLS achieve a BER close to 10^{-4}.

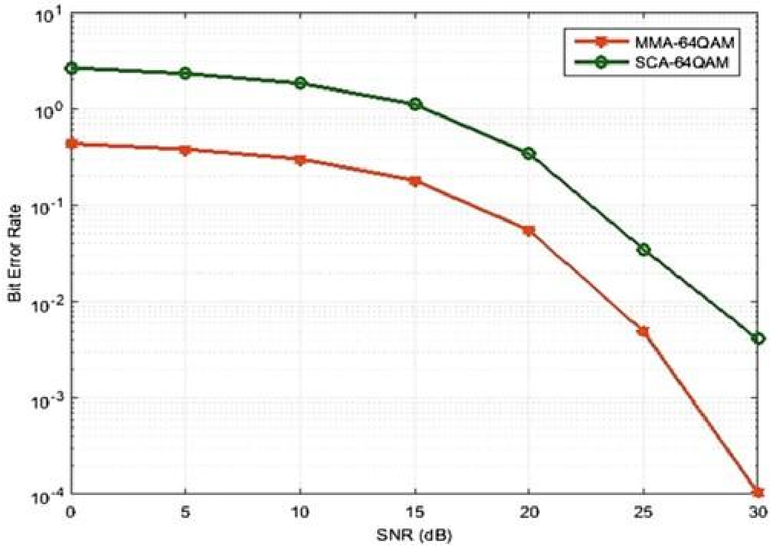

Figure 6: BER comparison of MMA and SCA algorithm for a 64 QAM constellation[22].

Figure 6 compares the BER performance of MMA and SCA for a 64-QAM modulation scheme. The MMA algorithm shows a consistent reduction in BER, reaching below 10^{-4} at an SNR of 30 dB.

Figure 7 compares the BER performance ofLMS and RLS for a 64-QAM modulation scheme. By the time the SNR reaches 25 dB, the LMS and RLS achieve a BER below to 10^{-3}.

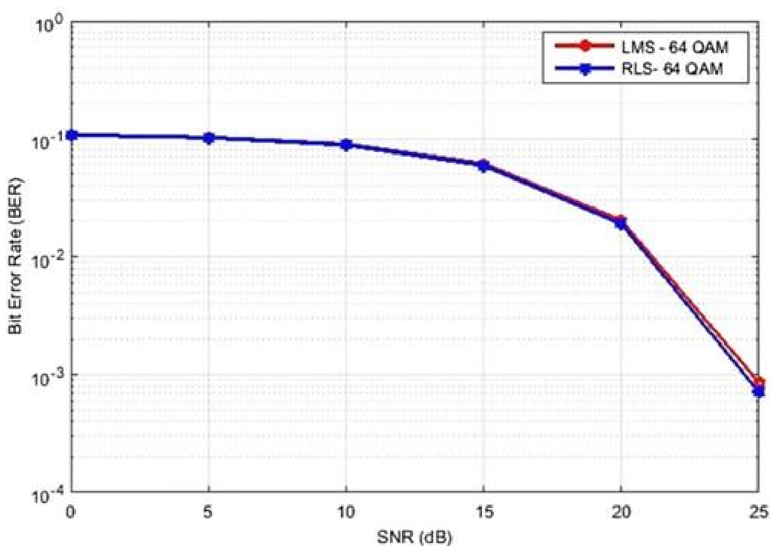

Figure 7: BER comparison of LMS and RLS algorithm for a 64-QAM constellation[22].

When comparing the curves MMA, SCA, LMS, and RLS, to the blue curve (Advanced MMA), it's evident that the proposed MMA algorithm demonstrates excellent performance for all order modulation schemes, the proposed MMA algorithm advantage becomes clear as we move towards more complex modulations like 64-QAM.

For example, at an SNR of 28 dB, the MMA-64QAM achieves a BER close to 10^{-4}, whereas the proposed MMA algorithm for 64-QAM achieves a BER of 10^{-6} .This difference emphasizes the effectiveness of the proposed MMA algorithm in maintaining low error rates in challenging conditions, making it a preferable choice for scenarios requiring higher data throughput and greater efficiency in 5G/6G mobile communications.

5. Conclusion

The unprecedented demands of next-generation wireless communication systems, particularly 5G and the forthcoming 6G networks, require innovative solutions to address the challenges of high data rates, spectral efficiency, and reliable signal recovery in the presence of noise and interference. QAM, with its ability to encode multiple bits per symbol, has become a cornerstone of these systems. However, the adoption of high-order QAM constellations (e.g., 64-QAM and 256-QAM) introduces greater susceptibility to channel impairments such as inter-symbol interference (ISI), necessitating the development of advanced equalization techniques. The MMA has demonstrated significant potential due to its capacity to independently equalize the in-phase (I) and quadrature (Q) components of QAM signals. In this paper, we presented an enhanced MMA algorithm designed to optimize performance in 5G and 6G communication systems. The proposed algorithm introduces several key improvements, including dynamic step size adaptation,and an accurate delay evaluation mechanism. These enhancements address critical issues in traditional MMA approaches, offering faster convergence, reduced residual ISI, improved stability, and lower bit error rates (BER) under diverse channel conditions. Through extensive simulations, the proposed algorithm has been evaluated across various QAM modulation schemes (16-QAM, 64-QAM, and 256-QAM) in multi-path fading channels with additive white Gaussian noise (AWGN).The proposed algorithm's ability to combine efficiency, adaptability, and robustness positions it as a promising solution for modern 5G/6G communication challenges.

Although the proposed MMA algorithm represents a significant advance in blind equalization for QAM signals, further research is needed to explore its potential in even more demanding scenarios. For example, future work could focus on integration with artificial intelligence (AI) and application in THz bands.

Thus, the improved MMA algorithm proposed in this work represents a robust and scalable solution to the challenges of blind equalization in modern communication systems. Its integration into 5G and 6G networks will play a key role in realizing the full potential of these technologies, enabling a future of seamless, high-performance wireless communication.

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