| F. Hamoudi A. Chaghi H. Amimeur | J. Electrical Systems: special issue N° 2 (2010): 82-96 | JES | | |
|---------------------------------------|---|-------------------------------------|--|--|
| E. Merabet | Regular paper Sliding Mode Control of a Three- | Journal of Electrical Systems | | |
| | phase Three-Leg Voltage Source Inverter Based Four-Wire Shunt Active Power Filter | | | |

The present paper deals with the sliding mode control of a three-phase four-wire shunt active power filter SAPF, to improve phase-current waveform, neutral current mitigation and reactive power compensation in electric power distribution system. The sliding mode is formulated using elementary differential geometry, and then the control vector is deduced from the sliding surface accessibility using the Lyapunov stability. The algorithm used to establish the current references for the sliding mode controller is based on the extraction of the fundamental positivesequence of the load current considering disturbed main voltages. It will be seen that this method permits to synthesis the control vector with simple manner, and finally, the obtained simulation results confirm that the above objectives are satisfied.

Keywords: Sliding mode Control, Four-Wire Active Filter, Harmonics Current Compensation.

1. INTRODUCTION

The increased severity of harmonic pollution in power distribution network has attracted the attention to develop dynamic and adjustable solutions to the power quality problems giving rise to active filter [1]-[5]. Three-wire active filtering provides compensation of harmonics, reactive power, but can't compensates zero sequence components caused by single-phase non-linear loads inherently generate more harmonics than three-phase non-linear loads, for this reason, four-wire active filtering is recommended in distribution system [3]-[5].

After synthesizing the reference currents, the voltage source inverter must inject these components in the point of common coupling with minimum error and fast response, this objective requires an appropriate current control method, in this regard, two different approaches are generally adapted; the fixed frequency control and the variable structure control (VSC). The first one requires a linear approach of the system to synthesize the control law. This approach can be advantageous for the fixed frequency but it is not very adapted to multi-frequencies signals. In fact the proportional integral controllers performed traditionally for this approach are known for their handicap to regulate correctly alternating references.

The sliding mode control (SMC) which is derived from the theory of variable structure control introduced since long time [6], is a known discontinuous control technique which takes in account the time varying topology of the controlled system. Thus this technique is naturally suitable to control systems based on power electronics devices in general [7]-[9], and active filter as particular case of these systems [10]-[11]. It is characterized by simplicity implementation, high robustness in presence of uncertainly in system parameters, and fast response. For these reasons, the sliding mode control can be successfully applied to achieve harmonics references regulation.

The purpose of this paper is the application of the sliding mode control to a three-phase four-wire shunt active filter based on three-leg voltage source inverter. The layout of this paper is as follows:

The system description and the establishment of the differential equations of the system will make the object of the first part of this paper. After that, we will explain briefly the algorithm of the reference current computation. The sliding mode control to active filter current control is then described in detail, and finally simulation results considering load variation will be given to measure the performances and the validity of the proposed control.

2. SYSTEM DESCRIPTION AND MODELLING

Fig. 1(a) illustrates the basic compensation principle of the four-wire shunt active power filter SAPF. The power circuit is based on three-phase three-leg controlled current voltage source PWM inverter connected to the grid at the AC side through a passive filter and uses two cascade connected capacitors as voltage source at the DC side, with the midpoint connected to the neutral wire of the grid to compensate neutral load current i_{LN} . An unbalanced nonlinear load is considered as a polluting source that draws unbalanced and distorted currents i_{Labc} from the mains. The SAPF is controlled to inject compensated current vector i_{cabc} in the grid in order to achieve source currents i_{sabc} balanced, sinusoidal and in phase with the fundamental main voltages, with keeping the DC-link voltages V_{C1} and V_{C2} balanced and in an admissible range.

To establish the dynamic equations of the system, let suppose that the power switches S_k can be assumed ideals, then the output voltage for each phase k to neutral can be expressed as follows:

$$v_{ck} = d_k V_{C1} - \overline{d}_k V_{C2} \tag{1}$$

 d_k (k = a, b, c) are the PWM switching functions given by:

$$d_k = \frac{u_k + 1}{2} \tag{2}$$

Where u_k is associated to the power switch states as follows:

$$u_k = 1$$
 when S_k on, S'_k off
 $u_k = -1$ when S_k off, S'_k on

Replacing (2) in (1) the active filter voltage is then rewritten as follows:

$$v_{ck} = \frac{1}{2}u_{ck} \left(V_{c1} + V_{C2} \right) + \frac{1}{2} \left(V_{C1} - V_{C2} \right)$$
(3)

The DC bus voltages across the two capacitors, related to u_k and the active filter currents i_{ck} as follows:

$$\frac{dV_{C1}}{dt} = \frac{1}{2C} \left(\sum_{k=a,b,c} u_k i_{ck} + \sum_{k=a,b,c} i_{ck} \right)$$
(4)

$$\frac{dV_{C2}}{dt} = \frac{1}{2C} \left(\sum_{k=a,b,c} u_k i_{ck} - \sum_{k=a,b,c} i_{ck} \right)$$
(5)

From the Kirchooff's voltage law, the interaction between the voltage source inverter and the grid is described by following differential equation

$$L_{c} \frac{di_{ck}}{dt} = -r_{c}i_{ck} + \frac{1}{2}u_{k}V_{dc} + \frac{1}{2}(V_{C1} - V_{C2}) - e_{k}$$
(6)

Where e_k represent the main voltages at the point of common coupling.

Finally, these equations are rearranged under matrix from:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} + \mathbf{e} \tag{7}$$



Fig. 1 Four-wire shunt active filter scheme.

| $\mathbf{e} = \begin{bmatrix} -\frac{e_a}{L_c} & -\frac{e_b}{L_c} & -\frac{e_c}{L_c} & 0 & 0 \end{bmatrix}^T$ $\mathbf{A} = \begin{bmatrix} -\frac{r_c}{L_c} & 0 & 0 & \frac{1}{2L_c} & -\frac{1}{2L_c} \\ 0 & -\frac{r_c}{L_c} & 0 & \frac{1}{2L_c} & -\frac{1}{2L_c} \\ 0 & 0 & -\frac{r_c}{L_c} & \frac{1}{2L_c} & -\frac{1}{2L_c} \\ \frac{1}{2C} & \frac{1}{2C} & \frac{1}{2C} & 0 & 0 \\ -\frac{1}{2C} & -\frac{1}{2C} & -\frac{1}{2C} & 0 & 0 \end{bmatrix}, \mathbf{B}(\mathbf{x}) + \begin{bmatrix} \frac{V_{dc}}{2L_c} & 0 & 0 \\ 0 & \frac{V_{dc}}{2L_c} & 0 \\ 0 & 0 & \frac{V_{dc}}{2L_c} \\ \frac{i_{ca}}{2C} & \frac{i_{cb}}{2C} & \frac{i_{cc}}{2C} \\ \frac{i_{ca}}{2C} & \frac{i_{cb}}{2C} & \frac{i_{cc}}{2C} \end{bmatrix}$ | When | \mathbf{x} = | $i_{ca} i_{cb} i_{c}$ | $x_{c} V_{C1} V$ | V_{C2} , $[1,,]$ | $\mathbf{u} = \left[u_a \right] u$ | $u_b u_c^{\mu}$, | | | |
|--|----------------|---|---|---|--|---|--------------------------|--|---|---|
| $\mathbf{A} = \begin{bmatrix} -\frac{r_c}{L_c} & 0 & 0 & \frac{1}{2L_c} & -\frac{1}{2L_c} \\ 0 & -\frac{r_c}{L_c} & 0 & \frac{1}{2L_c} & -\frac{1}{2L_c} \\ 0 & 0 & -\frac{r_c}{L_c} & \frac{1}{2L_c} & -\frac{1}{2L_c} \\ \frac{1}{2C} & \frac{1}{2C} & \frac{1}{2C} & 0 & 0 \\ -\frac{1}{2C} & -\frac{1}{2C} & -\frac{1}{2C} & 0 & 0 \end{bmatrix}, \ \mathbf{B}(\mathbf{x}) + \begin{bmatrix} \frac{V_{dc}}{2L_c} & 0 & 0 \\ 0 & \frac{V_{dc}}{2L_c} & 0 \\ 0 & 0 & \frac{V_{dc}}{2L_c} \\ \frac{i_{ca}}{2C} & \frac{i_{cb}}{2C} & \frac{i_{cc}}{2C} \\ \frac{i_{ca}}{2C} & \frac{i_{cb}}{2C} & \frac{i_{cc}}{2C} \\ \frac{i_{ca}}{2C} & \frac{i_{cb}}{2C} & \frac{i_{cc}}{2C} \\ \end{bmatrix}$ | e = [| $-\frac{e_a}{L_c}$ - | $-\frac{e_b}{L_c}$ $-\frac{e_b}{L_c}$ | $\frac{e_c}{L_c} = 0 \ 0$ | $) \end{bmatrix}^T$ | | | | | |
| | $\mathbf{A} =$ | $\begin{bmatrix} -\frac{r_c}{L_c} \\ 0 \\ 0 \\ \frac{1}{2C} \\ -\frac{1}{2C} \end{bmatrix}$ | 0 $-\frac{r_c}{L_c}$ 0 $\frac{1}{2C}$ $-\frac{1}{2C}$ | 0 $-\frac{r_c}{L_c}$ $\frac{1}{2C}$ $-\frac{1}{2C}$ | $ \frac{\frac{1}{2L_c}}{\frac{1}{2L_c}} $ $ \frac{1}{2L_c} $ $ 0 $ $ 0 $ | $-\frac{1}{2L_c} \\ -\frac{1}{2L_c} \\ -\frac{1}{2L_c} \\ 0 \\ 0 \end{bmatrix}$ | , B (x)+ | $\begin{bmatrix} V_{dc} \\ 2L_c \\ 0 \\ 0 \\ \frac{i_{ca}}{2C} \\ \frac{i_{ca}}{2C} \end{bmatrix}$ | 0 $\frac{V_{dc}}{2L_c}$ 0 $\frac{i_{cb}}{2C}$ $\frac{i_{cb}}{2C}$ | 0 $\frac{V_{dc}}{2L_c}$ $\frac{i_{cc}}{2C}$ $\frac{i_{cc}}{2C}$ |

3. ACTIVE FILTER CONTROL

_

In four-wire systems the current drawn by an unbalanced nonlinear load contains positive-sequence, negative-sequence and zero-sequence harmonics. In addition, if the main voltages are supposed unbalanced and contain harmonics, then the instantaneous real, imaginary and zero-sequence powers absorbed by the non linear load result from the different interactions between the different harmonics sequences of the load currents and main voltages. In this way, the constant real power can be expressed as follows [12]:

$$\overline{p}_{L} = \sum_{i=1}^{n} 3E_{i}^{+}I_{Li}^{+}\cos(\phi_{E_{i}^{+}} - \phi_{I_{Li}^{+}}) + \sum_{i=1}^{n} 3E_{i}^{-}I_{Li}^{-}\cos(\phi_{E_{i}^{-}} - \phi_{I_{Li}^{-}})$$
(8)

This can be rewritten as follows:

$$\overline{p}_{L} = 3E_{1}^{+}I_{L1}^{+}\cos(\phi_{E_{1}^{+}} - \phi_{I_{L1}^{+}}) + \left(\sum_{i=2}^{n} 3E_{i}^{+}I_{Li}^{+}\cos(\phi_{E_{i}^{+}} - \phi_{I_{Li}^{+}}) + \sum_{i=1}^{n} 3E_{i}^{-}I_{Li}^{-}\cos(\phi_{E_{i}^{-}} - \phi_{I_{Li}^{-}})\right) = \overline{p}_{Lf} + \overline{p}_{Lh}$$

$$(9)$$

Where, E_i^+ , E_i^- and I_{Li}^+ , I_{Li}^- are the rms values of the positive and negative sequences of the voltage and load current component for the i^{th} harmonic, whereas $\phi_{E_i^+}$, $\phi_{E_i^-}$ and $\phi_{I_{Li}^+}$, $\phi_{I_{Li}^-}$ are their phase shift respectively.

Equation (9) shows that if the SAPF is controlled to provide constant real power \overline{p}_L drawn from the source, then the source currents remain non sinusoidal because positive and negative sequences of the current that interact with the same sequences at the same frequencies will contribute also to a constant real power exchange \overline{p}_{Lh} , consequently, they are not seen as undesirable components, thus non compensated. As a conclusion, to guarantee sinusoidal source current, only \overline{p}_{Lf} must be delivered by the source.

3.1. Source Current Estimation

If only fundamental real power of the load is drawn from the source after compensation, then the source current contains only fundamental positive-sequence of the load current I_{L1}^+ . However, in the active filter operation, there are some active losses in the power switches and passive filter that cause variations in the DC bus voltage, therefore, to avoid this situation these losses must be compensated. This can be made by drawing an additional active current I_{loss} from the AC source. To achieve this operation a PI controller is traditionally used to generate this current from the error between the reference value V_{dc}^* and the measured value V_{dc} as follows:

$$I_{loss} = k_p \left(V_{dc}^* - V_{dc} \right) + k_i \int \left(V_{dc}^* - V_{dc} \right)$$
(10)

Thus, the peak value of the source current is:

$$I_{sp} = I_{Lp1}^{+} \cos(\phi_{E_{1}^{+}} - \phi_{I_{11}^{+}}) + I_{loss}$$
(11)

The three-phase source currents are then determined by detecting the fundamental pulsation of the main voltage, which is achieved using a phase locked loop. Otherwise to achieve source current in phase with the corresponding fundamental voltages, a phase shift detector based on Fourier analysis is used to extract the fundamental phase shift of each phase in the main voltages. Hence, the instantaneous source currents are given as:

$$i_{sa} = I_{sp} \sin(\omega t + \delta_{a1})$$

$$i_{sb} = I_{sp} \sin(\omega t + \delta_{b1})$$

$$i_{sc} = I_{sp} \sin(\omega t + \delta_{c1})$$
(12)

Where, δ_{a1} , δ_{b1} and δ_{c1} are the phase shifts of the fundamental main voltages e_{a1} , e_{b1} and e_{c1} respectively equal to 0, $-\frac{2\pi}{3}$ and $\frac{2\pi}{3}$ if there is no unbalance in the main voltages.

3.2. Compensation of the DC Bus Unbalance

A second loop is needed in the DC bus control to compensate the variations between V_{C1} and V_{C2} . Its role consists to force the SAPF to absorb a small DC component from the AC source such that if the average capacitor voltage V_{C1} is greater than V_{C2} , a negative DC-term current is added to the line current to compensate capacitor C_2 . Conversely, if the average capacitor voltage V_{C2} is greater than V_{C1} , a positive DC-term current is added to the line current to compensate capacitor C_1 . The compensating current can be computed directly from the difference $V_{C2} - V_{C1}$ as follows [13]:

$$I_{dc} = K (V_{C2} - V_{C1})$$
(13)

With K, a gain chosen small to avoid large DC term in the source current. Hence the instantaneous reference source currents are:

$$i_{sa}^{*} = I_{sp} \sin(\omega t + \delta_{a1}) + I_{dc}$$

$$i_{sb}^{*} = I_{sp} \sin(\omega t + \delta_{b1}) + I_{dc}$$

$$i_{sc}^{*} = I_{sp} \sin(\omega t + \delta_{c1}) + I_{dc}$$
(14)

Finally the compensating currents can be obtained from the reference source current and the load currents as follows:

$$i_{ca}^{*} = i_{sa}^{*} - i_{La}$$

$$i_{cb}^{*} = i_{sb}^{*} - i_{Lb}$$

$$i_{cc}^{*} = i_{sc}^{*} - i_{Lc}$$
(15)

The complete block diagram of the SAPF is shown in Fig. 2.

4. SLIDING MODE CONTROL

The sliding mode control consists to select the suitable switching configuration of the VSI in order to guarantee the state trajectory attraction toward a predefined sliding surface, and to maintain it stable over this surface.



Fig. 2 Complete block diagram of the proposed control.

The system established in (7) is a multi-input multi-output non-linear system. In order to formulate the sliding mode creation problem, letting:

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T$$
$$= \begin{bmatrix} i_{ca} & i_{cb} & i_{cc} & V_{C1} & V_{C2} \end{bmatrix}^T$$

Then the state equation (7) can be rearranged in the form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \tag{16}$$

Where the *n* dimensional vector field $\mathbf{f}(\mathbf{x})$, the $n \times m$ dimensional input matrix $\mathbf{G}(\mathbf{x})$ are given as follows:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -\frac{r_c}{L_c} x_1 + \frac{1}{2L_c} x_4 - \frac{1}{2L_c} x_5 - \frac{e_a}{L_c} \\ -\frac{r_c}{L_c} x_2 + \frac{1}{2L_c} x_4 - \frac{1}{2L_c} x_5 - \frac{e_b}{L_c} \\ -\frac{r_c}{L_c} x_3 + \frac{1}{2L_c} x_4 - \frac{1}{2L_c} x_5 - \frac{e_c}{L_c} \\ \frac{1}{2C} x_1 + \frac{1}{2C} x_2 + \frac{1}{2C} x_3 \\ -\frac{1}{2C} x_1 - \frac{1}{2C} x_2 - \frac{1}{2C} x_3 \end{bmatrix},$$

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} \frac{x_4 + x_5}{2L_c} & 0 & 0 \\ 0 & \frac{x_4 + x_5}{2L_c} & 0 \\ 0 & 0 & \frac{x_4 + x_5}{2L_c} \\ \frac{x_1}{2C} & \frac{x_2}{2C} & \frac{x_3}{2C} \\ \frac{x_1}{2C} & \frac{x_2}{2C} & \frac{x_3}{2C} \end{bmatrix}$$

4.1 Sliding Surfaces

For the *n* dimensional controlled system regulated by *m* independent switches, *m* sliding surface coordinate functions are defined. The *m* sliding surfaces are represented by the smooth algebraic restrictions $\sigma_i(\mathbf{x}) = 0$, i = 1, 2, ..., m. For each surface S_i , we have:

$$S_i = \left\{ \mathbf{x} \in \mathbb{R}^n \, \middle| \, \sigma_i(\mathbf{x}) = 0 \right\} \tag{17}$$

And the intersection of the m surfaces is denoted by S which verify:

$$S = \left\{ \mathbf{x} \in \mathbb{R}^n \, \middle| \, \mathbf{x} \in S_i, i = 1, 2, ..., m \right\}$$
(18)

Let define the vector of the sliding surface coordinate functions as follows:

$$\boldsymbol{\sigma}(\mathbf{x}) = \begin{bmatrix} \sigma_1(\mathbf{x}) \\ \sigma_2(\mathbf{x}) \\ \sigma_3(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \\ x_3 - x_3^* \end{bmatrix}$$
(19)

We know yet that when the sliding mode is reached, in other words, when the state vector is forced to evolve on the intersection of the sliding surface, i.e. $\mathbf{x} \in S$, the sliding surface coordinate function $\sigma(\mathbf{x})$ must satisfy the following condition:

$$(\dot{\sigma}(\mathbf{x}), \sigma(\mathbf{x})) = (\mathbf{0}, \mathbf{0})$$
 (20)

Then, we can write:

$$\dot{\boldsymbol{\sigma}}(\mathbf{x}) = \frac{\partial \boldsymbol{\sigma}(\mathbf{x})}{\partial \mathbf{x}^{T}} \left(\mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x}) \mathbf{u}_{eq}(\mathbf{x}) \right) = \mathbf{0}$$
(21)

We denote $\frac{\partial \boldsymbol{\sigma}(\mathbf{x})}{\partial \mathbf{x}^T} \mathbf{f}(\mathbf{x})$ by $L_f \boldsymbol{\sigma}(\mathbf{x})$, a *m* dimensional vector which represents the directional derivative of $\boldsymbol{\sigma}(\mathbf{x})$ along the direction of the vector field $\mathbf{f}(\mathbf{x})$. Similarly, the $m \times m$ dimensional matrix $\frac{\partial \boldsymbol{\sigma}(\mathbf{x})}{\partial \mathbf{x}^T} \mathbf{G}(\mathbf{x})$ is denoted by $L_G \boldsymbol{\sigma}(\mathbf{x})$. Thus, (21) is rewritten as follows: $\dot{\boldsymbol{\sigma}}(\mathbf{x}) = L_f \boldsymbol{\sigma}(\mathbf{x}) + L_G \boldsymbol{\sigma}(\mathbf{x}) \mathbf{u}_{eq}(\mathbf{x}) = \mathbf{0}$ (22)

This permits to define the equivalent control in the form:

$$\mathbf{u}_{eq}(\mathbf{x}) = -(L_G \boldsymbol{\sigma}(\mathbf{x}))^{-1} L_f \boldsymbol{\sigma}(\mathbf{x})$$
(23)

This means that as a condition for the equivalent control definition is that the matrix $L_G \sigma(\mathbf{x})$ must be invertible. Note also that the equivalent control must satisfy $-1 \le \mathbf{u}_{eq}(\mathbf{x}) \le 1$ which the necessary and sufficient condition for the sliding mode existence over the surface *S*.

Since the sliding mode is reached the state equation of the system is expressed as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) - \mathbf{G}(\mathbf{x}) (L_G \boldsymbol{\sigma}(\mathbf{x}))^{-1} L_f \boldsymbol{\sigma}(\mathbf{x})$$
(24)

4.2. Sliding Surface Accessibility

Let consider the following Lyapunov function:

$$V(\mathbf{\sigma}(\mathbf{x})) = \frac{1}{2} \mathbf{\sigma}^{T}(\mathbf{x}) \mathbf{\sigma}(\mathbf{x})$$
(25)

It is a semi-definite function, it is identically zero over the surface *S*, i.e. when $\sigma(\mathbf{x}) = \mathbf{0}$ and positive when $\sigma(\mathbf{x}) \neq \mathbf{0}$. The quantity $V(\sigma(\mathbf{x}))$ can be interpreted as the distance from the position of the point \mathbf{x} in the state space to the desired surface *S*. Therefore, in order to satisfy the condition $\sigma(\mathbf{x}) = \mathbf{0}$, the discrete control \mathbf{u} must exercise a closing or opening action, which permits to decrease

the distance $V(\sigma(\mathbf{x}))$, this means that the variation of this function in the time must be strictly negative, then;

$$\frac{d}{dt} \left(V(\boldsymbol{\sigma}(\mathbf{x})) \right) = \boldsymbol{\sigma}^T \left(\mathbf{x} \right) \dot{\boldsymbol{\sigma}}(\mathbf{x}) < 0$$
(26)

This is the condition for the trajectory attraction toward the sliding surface.

Referring to (22) and (26), if $\sigma(\mathbf{x}) \neq \mathbf{0}$, replacing $\mathbf{u}_{eq}(\mathbf{x})$ by \mathbf{u} , then the time derivative of the Lyapunov function can be expressed as follows:

$$\dot{V}(\boldsymbol{\sigma}(\mathbf{x})) = \boldsymbol{\sigma}^{T}(\mathbf{x}) \left(L_{f} \boldsymbol{\sigma}(\mathbf{x}) + L_{G} \boldsymbol{\sigma}(\mathbf{x}) \mathbf{u} \right) < 0$$
⁽²⁷⁾

Likewise, if $\sigma(\mathbf{x}) = \mathbf{0}$, then:

$$\dot{V}(\boldsymbol{\sigma}(\mathbf{x})) = \boldsymbol{\sigma}^{T}(\mathbf{x}) \left(L_{f} \boldsymbol{\sigma}(\mathbf{x}) + L_{G} \boldsymbol{\sigma}(\mathbf{x}) \mathbf{u}_{eq} \right) = 0$$
(28)

Now, if we consider that the switching frequency is infinite or sufficiently high, we can suppose with good approximation that the state vector \mathbf{x} takes the same value in the both case (27) and (28). Thus, subtracting (28) from (27), the restriction (26) can be rewritten as follows

$$\dot{V}(\boldsymbol{\sigma}(\mathbf{x})) = \boldsymbol{\sigma}^{T}(\mathbf{x})L_{G}\boldsymbol{\sigma}(\mathbf{x})(\mathbf{u} - \mathbf{u}_{eq}(\mathbf{x})) < 0$$
⁽²⁹⁾

This inequality can be achieved by applying the control vector given by:

$$\mathbf{u} = -sign\left(\boldsymbol{\sigma}^{T}\left(\mathbf{x}\right)L_{G}\boldsymbol{\sigma}\left(\mathbf{x}\right)\right)^{T}$$
(30)

And finally, the switch position functions:

$$\mathbf{d} = \frac{1}{2} \left(\mathbf{1} + \mathbf{u} \right) = \frac{1}{2} \left(\mathbf{1} - sign \left(\boldsymbol{\sigma}^T \left(\mathbf{x} \right) L_G \boldsymbol{\sigma} \left(\mathbf{x} \right) \right)^T \right)$$
(31)

5. SIMULATION RESULTS

The performances of the developed sliding mode control were verified through simulation using MATLAB software. The polluting load is constituted with three-phase thyristor rectifier, single-phase thyristor rectifier and single-phase diode rectifier. A detailed method for active filter parameters optimization is presented in [14] hence, the main parameters of the simulated system are in Table 1.

TABLE I

| Phase to neutral voltage source | 230Vrms, 50Hz |
|---------------------------------|----------------------|
| Source inductance | $L_s = 100 \mu H$ |
| DC-bus capacitors | $C_1 = C_2 = 5mF$ |
| DC-bus voltage reference | $V_{dc}^{*} = 1000V$ |
| Inductor filter | $L_c = 2mH$ |

MAIN PARAMETERS OF THE SIMULATED CIRCUIT

First, in Fig. 3 and Fig. 4 the three-phase and neutral currents are represented respectively with and without active filtering, with illustrating the harmonic spectrums for each phase. It can be seen from Fig. 5, the current drawn by the load is unbalanced and contain positive-sequence harmonics $6h + 1(7^{\text{th}}, 13^{\text{th}}...)$, negative sequence harmonics $6h + 5(5^{\text{th}}, 11^{\text{th}}...)$, and zero-sequence harmonics $6h + 3(3^{\text{rd}}, 9^{\text{th}}...)$. With *h* indicates the harmonic order. Note that the zero-sequence harmonics is the result of the 4th-wire (neutral), this sequence does not appears in the three-wire systems.

With introducing active filtering action, the harmonic distortions for the compensated components are all less than 1.2% and almost only the fundamental of the positive-sequence remains, therefore the neutral current is cancelled. Table 2 summarizes the detailed results of current compensation.

Figure 6 illustrates the performances of the active filter controller to guarantee source current in sinusoidal form and in phase with main voltage, under load change, where it can be seen that the power factor and the total harmonic distortion are both excellently improved before and after load change.

The DC-bus voltage regulation is shown in Figure 7, the DC-bus is charged and the voltage level V_{dc} reaches the reference value V_{dc}^* with excellent response (950V < V_{dc} < 1050V in 0.05s). The excellent dynamic performance is also observed with load change operated at t = 0.2s, in fact the DC-link voltage is remained inside the bounded range [950V, 1050V] and stabilized at V_{dc}^* in 0.1s. The DC-link unbalance is shown in the same figure where one can remark that the two voltages V_{C1} and V_{C2} are sufficiently balanced, accept the existence of small oscillation ΔV_{dc} (about 1.3% of V_{dc}) which can be neglected. It should be noted that the DC-link voltage oscillations are strongly depending of the capacitors value. In fact, it is known that to keep these oscillation inside an acceptable bounded ranges, generally large capacitors are recommended, for this reason, in this paper this parameters is relatively large.



Fig. 3 AC mains three-phase and neutral currents without active filtering.



Fig. 4 AC mains three-phase and neutral currents with active filtering.



Fig. 5 Harmonic spectrums of the load and source phase currents.



Fig. 6 Harmonics and reactive power compensation under load change, for the case of *a*-phase

| 3- | Load | Source | Load | Source |
|------------------|---------|--------|--------------|--------|
| phase neutral | THD (%) | | Fund RMS (A) | |
| a – ph | 28.29 | 01.70 | 54.03 | 54.42 |
| <i>b</i> – ph | 27.67 | 01.48 | 58.74 | 54.53 |
| c-ph | 23.76 | 01.76 | 65.21 | 54.50 |
| Neutral | 31.31 | | 13.10 | |

 TABLE II

 DETAILED RESULTS FOR THREE-PHASE AND

 NEUTRAL CURRENTS COMPENSATION.



6. CONCLUSION

In this paper, a sliding mode control for three-phase three-leg voltage source inverter based four-wire shunt active filter is applied. The system is observed as a MIMO-decoupled nonlinear system, which permits a simple implementation. The simulation results show the ability of this method to track references with minimum error, fast response and high robustness. A low line current THD, and high power factor are provided even load change. Finally, as critics, it should be noted that, the decoupled nature of the four-wire three-leg VSI studied in this paper is simple to analyze but we have seen that it need very large capacitors value in the DC-bus. For the sliding mode control, it is known that the switching frequency presents its principal incontinent. The replacement of the three-leg topology by the four-leg one can resolve the problem of first mentioned critic, and we know yet that the switching frequency stabilization is possible. This will make the object of the continuation for this work.

REFERENCES

- [1] H. Akagi, "New Trends in Active Filter for Power Conditioning". *IEEE Transa.on Industry Applications*, Vol. 32, No. 6, pp. 1312-1322, December 1996.
- [2] Singh, K. Al-Haddad and A. Chandra, "A Review of Active Filters for Power Quality Improvement". *IEEE Trans. on Industrial Electronics*, Vol. 46, No. 5, pp. 960-971, October 1999.
- [3] M. Aredes, J. Häfner, K. Heulmann, "Three-Phase Four-Wire Shunt Active Filter Control Strategies" *IEEE Trans. on Power Electronics*, Vol. 12, No. 2, pp. 311-318, March 1997.
- [4] B. Singh, K. Al-Haddad, A. Chandra, "Harmonic Elimination, Reactive Power Compensation and Load Balancing in Three-Phase, Four-Wire Electric Distribution Systems Supplying Non-Linear Loads". *Electric Power Systems Research* 44, pp. 93-100, 1998.
- [5] M. Ucar, E. Ozdemir, "Control of a 3-Phase 4-Leg active Power Filter under Non-Ideal Mains Voltage Condition", *Electric Power Systems Research 2007*.
- [6] V. A. Utkin, "Variable Structure Systems with Sliding Mode", *IEEE Trans. on Automatic Control*, Vol. AC22, No. 2, pp. 1105-1120, 1997.
- [7] G. Spiazzi, P. Mattavelli, L. Rossetto, L. Alesani, "Application of Sliding Mode Control to Switch-Mode Power Supplies", *Journal of Circuits, Systems and Computers (JCSC)*, Vol. 5, No. 3, pp. 337-354, September 1995.
- [8] M. Ahmed, "Sliding Mode Control for Switched Mode Power Supplies", PhD. Thesis, Lappeenranta University of Technology, Finland, 2004.
- [9] Hebertt Sira-Ramirez and Ramon Silva-Ortigoza, "Control Design Techniques in Power Electronics Devices", Springer-Verlag, London, 2006.
- [10] N. Sabanovic, T. Ninomiya, A. Sabanovic, B. Perunicic, "Control of Three-Phase Switching Converters: A Sliding Mode Approach", *PESC*'93, pp. 630-635, 1993.
- [11] N. Mendalek, K. Al-Haddad, F. Fnaiech, and L. A. Dessaint, "Sliding Mode Control of 3- Phase 3-Wire Shunt Active Filter in the dq Frame", *IEEE CCECE'2001*, pp. 765-770, 2001.
- [12] M. Aredes and E. H. Watanabel, "New Control Algorithms for Series and Shunt Three-Phase Four-Wire Active Power Filters" *IEEE Trans. Power delivery*, Vol. 10, No. 3, july 1995 pp1649-1656
- [13] Bor-Ren Lin and Tsung-Yu Yang, "Analysis and Implementation of Three-Phase Power Quality Compensator under the Balanced and Unbalanced Load Conditions", *Electric Power Systems Research* 76 (2006) 271–282
- [14] Alessandro Cavini, Fabio Ronchi, Andrea Tilli "Four-Wires Shunt Active Filters: Optimized Design Methodology". 0-7803-7906-3/03/17.00 02003 IEEE, pp 2288-2293, 2003.