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J. Electrical Systems 13-4 (2017): 742-751

Regular paper

A Three-phase Unsymmetrical Distribution Power Flow Solution Based on Symmetrical Component Theory

JES Journal of Electrical Systems

Aiming at the three-phase unsymmetrical distribution network, this paper proposed a three-sequence components decoupled power flow method based on the node-branch incidence matrix. The three-sequence decoupled models were established based on symmetrical component theory by utilizing the weakly coupling feature of three-phase unsymmetrical distribution networks. The algorithm simplifies the complexity of the distribution network and reduces the dimensions of the matrix, so the calculation process is relatively simple. Moreover, a new approach deal with the PV nodes was developed based on the assumption that the positive-sequence voltage magnitude of PV node is sustained at a given fixed value. The formula to calculate the reactive power increment for each PV node was derived based on Thevenin equivalent circuit theory. The test results show that the proposed method and PV nodes processing approach have better convergence and faster calculating speed.

Keywords: Electricity spot market; bilateral contracts; thermal unit commitment; mixed-integer linear programming.

Article history: Received 7 December 2015, Accepted 13 October 2017

1. Introduction

Compared to the transmission network, the distribution network are unsymmetrical, load-unbalanced and weakly-meshed, etc. These characteristics need to be considered in power flow algorithms. The existing power flow algorithms include the improved fast decoupled method, the implicit Zbus Gaussian method, the improved Newton method and the forward/backward sweep based method [1-4]. The forward/backward sweep based method has been widely used for its low storage space requirements, fast calculation speed, good convergence and easy programming.

Generally, the power flow of three-phase unbalanced distribution network can be calculated based on the three-phase models, such as the previous mentioned algorithms. Literature [5] realized the direct calculation of three-phase power flow by establishing the incidence formula between node voltages and injection currents based on the path matrix. Literature [6] presented a power flow algorithm based on injected current analysis for three-phase unbalanced distribution network. Another common method to calculate the power flow of distribution network is sequence-component method. Literature [7-9] utilized the sequence component method to solve the power flow. The three-sequence decoupled solution was realized by converting the sequence component into positive, negative and zero sequence networks. Compared to the direct solution, the three-sequence decoupled algorithm is simple, but its accuracy deteriorates.

The introduction of distributed generation (DG) into distribution power network has great influence to the power flow, voltage quality, etc. Especially the PV node has constant voltage amplitude, fixed active power, but unknown reactive power, so it can't be handled

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easily. The current compensation based methods for multi-port systems are used to solve radial and weakly meshed systems with PV nodes and have good efficiency [10-12]. But when network become more meshed and has more PV nodes, the converged iteration increases significantly and sometimes the results do not converge. An improved Newton-Raphson method to handle PV node was presented in [13], but the method was too complex and it needs to solve the Jacobian matrix. Literature [6] and [14] presented two methods to handle PV nodes and calculate the reactive power change with the path matrix based power flow method and achieved good results.

On the basis of the previous literatures and the weakly-coupled features among three-phase distribution networks, this paper developed a three-sequence components decoupled power flow method for the radial distribution network. Meanwhile, a new method to deal with PV nodes has been studied and developed. The feasibility and effectiveness of the method have been tested and compared based on practical examples.

2. Power flow algorithm for single-phase radial network

Assuming that the radial distribution network has N+1 nodes and the source node is used as the reference node. The number of independent nodes is N and the number of branches is equal to N too. This network can be described using the reduced node-branch incidence matrix $\mathbf{A}$ of $N \times N$. The element in $\mathbf{A}$ is,

$$a_{ij} = \begin{cases} -1 & \text{Node } i \text{ is the starting point of branch } j. \\ 1 & \text{Node } i \text{ is the ending point of branch } j. \\ 0 & \text{Node } i \text{ is neither the starting point nor the ending point of branch } j. \end{cases}$$

Each row in $\mathbf{A}$ corresponds to one node and indicates which branch connect to this node (not including the reference node). Each column in $\mathbf{A}$ corresponds to one branch. There are only two non-zero elements, “1” and “-1”, in each column, which give the starting node number and the ending node number of the branch. Let node injection branches not be included in the network and the node injection current is positive when it flows out of the node. Define branch current vector is $\boldsymbol{I}_{\mathrm{b}}$ ($N \times 1$) and node injection current vector is $\boldsymbol{I}_{\mathrm{g}}$ ($N \times 1$), then has:

$$\boldsymbol{I}_{\mathrm{g}} = \mathbf{A}\boldsymbol{I}_{\mathrm{b}} \Rightarrow \boldsymbol{I}_{\mathrm{b}} = \mathbf{A}^{-1}\boldsymbol{I}_{\mathrm{g}} = \mathbf{T}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{g}} \qquad (1)$$

Where $\mathbf{T}$ ($\mathbf{T}^{\mathrm{T}} = \mathbf{A}^{-1}$) is also called the path (path-branch incident) matrix based on the description of graph theory.

Let branch voltage vector be $\boldsymbol{U}_{\mathrm{b}}$ ($N \times 1$) and branch impedance diagonal matrix is $\boldsymbol{Z}_{\mathrm{b}}$ ($N \times N$), then:

$$\boldsymbol{U}_{\mathrm{b}} = \boldsymbol{Z}_{\mathrm{b}}\boldsymbol{I}_{\mathrm{b}} = \boldsymbol{Z}_{\mathrm{b}}\mathbf{T}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{g}} \qquad (2)$$

Let the voltage of reference node be $\dot{U}_{\mathrm{s}}$ and the node voltage vector be $\boldsymbol{U}_{\mathrm{n}}$ ($N \times 1$).The voltage difference between power supply and any other node i is equal to the sum of tree branch voltage along the path of node i, that is:

$$\Delta\dot{U}_{\mathrm{n}i} = \dot{U}_{\mathrm{s}} - \dot{U}_{\mathrm{n}i} = \mathbf{T}_i\boldsymbol{Z}_{\mathrm{b}}\boldsymbol{I}_{\mathrm{b}} \qquad (i = 1, 2, \cdots, N) \quad (3)$$

Where, $\mathbf{T}_i$ is the node i corresponding row vector（path vector）in $\mathbf{T}$.

Formula (3) is the key of power flow calculation, the steps are as follows:

Step 1: data preparation;

Step 2: S_i is the total power injection in node i, and Y_i is sum of all shunt admittance corresponding to node i, so:

$$\dot{I}_{gi} = (S_i/\dot{U}_{ni})^* \text{-} Y_i\dot{U}_{ni} \quad \left(i = 1,2,\cdots,N \right) \quad (4)$$

Step 3: calculate $\Delta\dot{U}_{ni}$ with (3), then $\dot{U}_{ni} = \dot{U}_s - \Delta\dot{U}_{ni}$;

Steps 1-3 are repeated until convergence is achieved (the maximum voltage mismatch is small enough).

3. Decoupled algorithm for three-phase unsymmetrical network

With the symmetrical component theory, the unbalanced three-phase currents and voltages can be decoupled into the superposition of zero, positive and negative sequence currents and voltages. And then the system can be studied with three single-phase networks as positive, negative and zero sequence networks. Let $a = e^{j2\pi/3}$, $\mathbf{H} = [1\ 1\ 1;\ 1\ a^2\ \ a;\ 1\ a\ \ a^2]$, then (0, 1 and 2 represent zero, positive and negative sequence respectively; a, b and c represent a-phase, b-phase and c-phase respectively):

$$\boldsymbol{U}_{012} = [\dot{U}_0\ \ \dot{U}_1\ \ \dot{U}_2]^{\mathrm{T}} = \mathbf{H}^{-1}\boldsymbol{U}_{\mathrm{p}} = \mathbf{H}^{-1}[\dot{U}_{\mathrm{a}}\ \ \dot{U}_{\mathrm{b}}\ \ \dot{U}_{\mathrm{c}}]^{\mathrm{T}} \qquad (5)$$

$$\boldsymbol{I}_{012} = [\dot{I}_0\ \ \dot{I}_1\ \ \dot{U}_2]^{\mathrm{T}} = \mathbf{H}^{-1}\boldsymbol{I}_{\mathrm{p}} = \mathbf{H}^{-1}[\dot{I}_{\mathrm{a}}\ \ \dot{I}_{\mathrm{b}}\ \ \dot{I}_{\mathrm{c}}]^{\mathrm{T}} \qquad (6)$$

Based on Ohm's law and (5) and (6), (7) can be obtained as

$$\boldsymbol{U}_{012} = \mathbf{H}^{-1}\boldsymbol{U}_{\mathrm{p}} = \mathbf{H}^{-1}\mathbf{Z}_{\mathrm{p}}\boldsymbol{I}_{\mathrm{p}} = \mathbf{H}^{-1}\mathbf{Z}_{\mathrm{p}}\mathbf{H}\mathbf{H}^{-1}\boldsymbol{I}_{\mathrm{p}} = \boldsymbol{Z}_{012}\boldsymbol{I}_{012} \qquad (7)$$

Here $\boldsymbol{Z}_{012} = \mathbf{H}^{-1}\mathbf{Z}_{\mathrm{p}}\mathbf{H}$. And $\mathbf{Z}_{\mathrm{p}} = [Z_{\mathrm{aa}}\ Z_{\mathrm{ab}}\ Z_{\mathrm{ac}};Z_{\mathrm{ba}}\ Z_{\mathrm{bb}}\ Z_{\mathrm{bc}};Z_{\mathrm{ca}}\ Z_{\mathrm{cb}}\ Z_{\mathrm{cc}}]$ is the unbalanced three-phase line impedance matrix, $\boldsymbol{Z}_{012}$ is three-sequence impedance matrix. When the spatial distribution of three-phase lines is symmetrical, all non-diagonal elements in $\boldsymbol{Z}_{012}$ are zero. When the spatial distribution is unsymmetrical, non-diagonal elements in $\boldsymbol{Z}_{012}$ are not zero which means that there's coupling among the sequence impedances, but generally, the absolute values of non-diagonal elements are far less than those of the diagonal elements and can be ignored. So $\boldsymbol{Z}_{012}$ can be simplified as

$$\boldsymbol{Z}_{012} = \mathrm{diag}[Z_0,\ Z_1,\ Z_2] \qquad (8)$$

For each sequence component can be calculated separately with (7) and (8), the simultaneous power flow calculation of the original three-phase distribution network is then converted into the calculation of three single-phase networks.

Let the source voltage be $\boldsymbol{U}_{\mathrm{sp}}$ (3×1), the node voltages be U_{np} (3n×1) and the node injection currents be $\boldsymbol{I}_{\mathrm{gp}}$ (3n×1) for the three-phase unsymmetrical distribution network, then the node injection phase current is calculated by,

$$\dot{I}_{\mathrm{gp}i} = (S_{\mathrm{p}i}/\dot{U}_{\mathrm{np}i})^* \text{-} Y_{\mathrm{p}i}\dot{U}_{\mathrm{np}i} \quad \left(i = 1,2,\cdots,N;\mathrm{p{=}a,b,c} \right)$$

The three-sequence components of $\boldsymbol{I}_{\mathrm{gp}}$ can be calculated with (6). Then, the zero, positive and negative sequence voltages can be obtained by using the single-phase power flow algorithm described in Section 1. Next, the three-phase voltage of each node can be calculated by the inverse transformation of (5) and the next iteration starts.

4. Handling of PV Nodes

PQ-type DGs can be easily coped with in power flow algorithm. But PV-type DGs have fixed values of active power but have unknown reactive power necessary to sustain the voltage at the scheduled voltage magnitude. So the key is to calculate and update the output reactive power increment of each PV node.

Here, the reactive power increment will be derived based on the assumption that the positive-sequence voltage magnitude of PV nodes is invariable and with the positive-sequence network. The Thevenin equivalent circuit from the breakpoint at PV node is shown in Fig.1. The open-circuit voltage $\dot{U}_{\mathrm{d}i}$ is the same as the calculated voltage of the connected node i for PV node. $\dot{U}_{\mathrm{ds}i}$ is the voltage of the PV node with fixed magnitude value but unknown angle value.

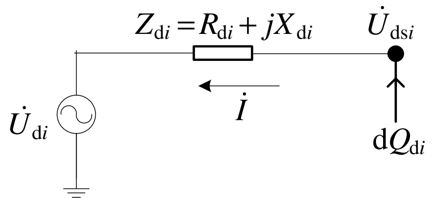

Fig. 1 the Thevenin equivalent circuit for PV node

$Z_{\mathrm{d}i}$ is the Thevenin equivalent impedance from the breakpoint in the positive-sequence network, which equals the summation of the branch impedances along the path of connected node i in $\mathbf{T}_{\mathrm{t}}$. Let $\boldsymbol{Z}_1$ be the branch positive sequence impedance diagonal matrix of $N\times N$, then [5]

$$Z_{\mathrm{d}i}=R_{\mathrm{d}i}+jX_{\mathrm{d}i}=\mathbf{T}_{\mathrm{i}}\boldsymbol{Z}_{\mathrm{b1}}\mathbf{T}_{\mathrm{i}}^{\mathrm{T}} \qquad (9)$$

The apparent power increment of PV node can be calculated by,

$$\mathrm{d}S=\dot{U}_{\mathrm{ds}i}\left(\frac{\dot{U}_{\mathrm{ds}i}-\dot{U}_{\mathrm{d}i}}{Z_{\mathrm{d}i}}\right)^{*}=\frac{U_{\mathrm{ds}i}^{2}}{\left|Z_{\mathrm{d}i}\right|}\angle\theta-\frac{U_{\mathrm{ds}i}U_{\mathrm{d}i}}{\left|Z_{\mathrm{d}i}\right|}\angle(\delta+\theta) \qquad (10)$$

Where $\delta=\angle\dot{U}_{\mathrm{ds}i}-\angle\dot{U}_{\mathrm{d}i}$ and $\theta=\angle Z_{\mathrm{d}i}$. Expanding (10), the output active and reactive power increment of the PV node are,

$$\mathrm{d}P=\left[U_{\mathrm{ds}i}^{2}\cos\theta-U_{\mathrm{ds}i}U_{\mathrm{d}i}\cos(\delta+\theta)\right]\Big/\left|Z_{\mathrm{d}i}\right| \qquad (11)$$

$$\mathrm{d}Q=\left[U_{\mathrm{ds}i}^{2}\sin\theta-U_{\mathrm{ds}i}U_{\mathrm{d}i}\sin(\delta+\theta)\right]\Big/\left|Z_{\mathrm{d}i}\right| \qquad (12)$$

The active power of PV node is fixed, that is $\mathrm{d}P=0$. Substituting it into (11) and rearranging (11) and (12) give,

$$\cos(\delta+\theta)=U_{\mathrm{ds}i}\cos\theta/U_{\mathrm{d}i} \qquad (13)$$

$$\sin(\delta+\theta)=U_{\mathrm{ds}i}\sin\theta/U_{\mathrm{d}i}-\mathrm{d}Q\left|Z_{\mathrm{d}i}\right|/(U_{\mathrm{ds}i}U_{\mathrm{d}i}) \qquad (14)$$

Squaring the both sides of (13) and (14) and then adding them together to eliminate $\delta+\theta$ gives,

$$\left(\frac{\left|Z_{\mathrm{d}i}\right|}{U_{\mathrm{ds}i}U_{\mathrm{d}i}}\right)^{2}(\mathrm{d}Q)^{2}-2\frac{\left|Z_{\mathrm{d}i}\right|\sin\theta}{U_{\mathrm{d}i}^{2}}\mathrm{d}Q+\frac{U_{\mathrm{ds}i}^{2}}{U_{\mathrm{d}i}^{2}}=1 \qquad (15)$$

And $\sin\theta=X_{\mathrm{d}i}\big/\left|Z_{\mathrm{d}i}\right|$, substituting it into (15) and rearranging it has

$$(\mathrm{d}Q)^{2}-2c_{1}\mathrm{d}Q+c_{2}=0 \qquad (16)$$

Where, $c_1=X_{\mathrm{d}i}U_{\mathrm{ds}i}^{2}\big/\left|Z_{\mathrm{d}i}\right|^{2}$, $c_2=U_{\mathrm{ds}i}^{2}(U_{\mathrm{ds}i}^{2}-U_{\mathrm{d}i}^{2})\big/\left|Z_{\mathrm{d}i}\right|^{2}$. Then, $\mathrm{d}Q$ can be gotten by solving (16). And for the less change of the output reactive power, the better, so,

$$\mathrm{d}Q=c_1-\sqrt{c_1^{2}-c_2} \qquad (17)$$

But note that the output reactive power Q in PV node is limited at each PV node, so if Q violates the upper or lower limit, it will be set to upper or lower limit and then the PV node will be converted to PQ node during next iteration.

5. Examples and analysis

5.1 Efficiency analysis of the proposed algorithm

A 6-Bus unbalanced test system in [5] and the 13-Bus, 37-Bus and 123-Bus unbalanced test systems in [15] are used to test the proposed method. The converged results are shown in Tab.1 based on algorithm in [5] and the algorithm proposed in this paper. The configuration of the used laptop is Pentium 1.73GHz Dual Core microprocessors with 1.5G memory.

Tab.1 Comparison of Converged Performance for test result

case	algorithm in literature [5]		algorithm in this paper	
	iterations	time/ms	iterations	time/ms
6Bus	5	13.0	6	9.7
13Bus	7	26	7	19
37Bus	7	50	7	37
123Bus	7	180	7	135

The calculation time is the average value of 500 times and convergence accuracy is 10^{-6} p.u. for the maximum mismatch of node voltages. It can be seen from Tab.1, the proposed three-sequence decoupled algorithm have the similar convergence with the algorithm in [5]. But the new method simplifies the complexity of the three-phase unsymmetrical distribution network and reduces the dimensions of the matrix, so the calculation time is significantly reduced.

Due to the weakly coupling among the three-phase unsymmetrical distribution networks, the results from the proposed method have errors (the converged voltages that solved with the loop theory based method in [5] are used as the benchmark). However, from the above calculation results, the voltage magnitude error is no more than 0.005(p.u.) . Since the loop theory based method in [5] has been generally acknowledged to be one of the best performing algorithm, the proposed method can be seen as being capable of offering an enough accuracy level for networks under good voltage conditions.

5.2 Cases studies when including PV nodes

5.2.1 Case I

The 6-Bus test systme introduced in [5] is shown in Fig.2. The transformer is of Y_n-Y_n connection and two DGs of PV type are connected to bus 3 and bus 5. The active power of PV1 and PV2 are both set to 250kW. The output reactive powers for PV1 and PV2 are assumed unlimited. The convergence accuracy is 10^{-6} p.u. for node voltages and the threshold of PV node voltage mismatch is 0.01 p.u. And at the beginning of the first iteration, the initial value of output reactive power at each PV node could be set to 0.

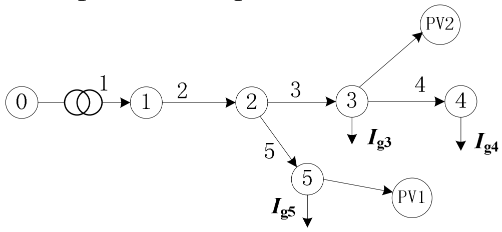

Fig.2 6 bus system with two PVs

The proposed power flow in this paper is used to calculate the power flow. And in order to deal with PV node, the proposed approach in this paper (new approach), the approach in [5] (approach 1) and the approach in [14] (approach 2) are adopted. The results are shown in Tab.2.

Tab.2 Converged solutions for 6 bus test system

PVs	approach	iterations	time /ms	positive-sequence voltage of PV nodes/p.u.	output reactive power of PV nodes/kVar
PV1	new approach	5	14	0.999 0	541.2
	approach 1	5	16	0.998 7	533.6
	approach 2	5	11	0.997 1	549.1
PV1 ~2	new approach	5	19	0.994 7, 0.992 6	166.1, 416.4
	approach 1	5	22	0.999 2, 0.998 8	202.7, 535.0
	approach 2	5	14	0.998 7, 0.997 4	209.3, 549.9

From Tab.2 can be seen, the convergence performance of the three approaches is basically the same. The approach 2 has the least calculation time and the approach 1 has the most calculation time. For only one PV (PV1), the output reactive power and the positive-sequence voltage at PV1 node are basically the same. But careful contrast shows that approach 2 has a larger reactive power output but with a relatively poor voltage compared with the results from new approach and approach 1. For PV1~2, the reactive power output of new approach proposed in this paper is much smaller than the other two approaches (total 155.2kVar less than approach 1 and 176.8kVar less than approach 2). PV plays a very good improvement to update the node voltage, and the more PVs nodes the better. However, the actual PV nodes have limited reactive power output, it may be impossible to achieve such good results.

5.2.2 Case II

The modified 69Bus system described in [5] is shown in Fig.3, The line space is symmetrical and the three-phase loads are unbalanced for this test system. Six PV nodes are added at node 88, 46, 34, 52, 14 and 23 respectively. Their rated active powers are 220kW, 300kW, 250kW, 300kW, 200kW and 250kW in turn. The lower and upper limits of output reactive power at each PV node are set to zero and the value of the rated active power. The convergence conditions are the same with case I. Three conditions were discussed: 1) Condition 1 with PV1~2; 2) Condition 2 with PV1~4; 3) Condition 3 with PV1~6. The converged calculation results are shown in Tab.3 by using the above three approaches.

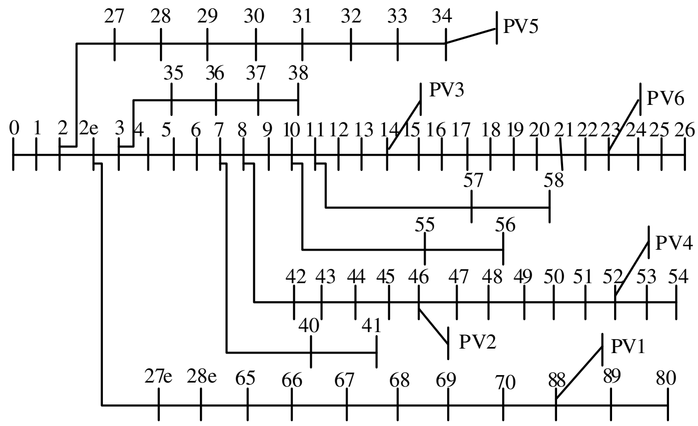

Fig.3 Modified sixty-nine buses system with six PV nodes

As shown in Tab.3, when the output reactive powers are limited, the positive-sequence voltage for some PV node cannot meet the requirements and the PV node is converted into PQ node. For CASE1~3, the results from the three approaches to calculate the output reactive powers and positive-sequence voltages at PV nodes are basically the same. But compared to approach 1 and approach 2, the new approach has better convergence performance and less iteration number. The new algorithm has the least calculation time and the calculation time increases less and remains almost constant with more PVs.

From the voltage amplitude profile for three cases and no PV, the node voltage in some nodes are very low when no PV and are even lower than 0.95 (p.u.) which does not meet the requirements. But in case1~3, the voltage at each node has been improved. The voltages at all nodes are more than 0.96 (p.u.) with four PVs (CASE2). Also, the voltages are better with more PVs.

Tab.3 Converged solutions of for 69 bus test system

PV input	approach	iteration	time (ms)	positive-sequence voltage of PV nodes/p.u.	reactive power output of PV nodes /kVar
0		6	61		
Condition 1	new approach	6	75	1, 0.962 1	12.4, 300
	approach 1	6	89	1, 0.962 1	12.1, 300
	approach 2	6	135	1, 0.962 1	12.4, 300
Condition 2	new approach	6	79	1, 0.962 1 0.979 2, 0.9594	11.4, 300, 300, 250
	approach 1	7	124	1, 0.962 1 0.979 2, 0.9594	11.1, 300, 300, 250
	approach 2	7	158	1, 0.962 1, 0.979 2, 0.9594	11.4, 300, 300, 250
Condition 3	new approach	5	72	1, 0.976 7, 0.991 8, 0.961 8, 1.00 5, 0.992 7	10.8, 300, 300, 250, 0, 250
	approach 1	7	145	1, 0.976 7, 0.991 9, 0.961 8,1.00 5, 0.992 7	11.1, 300, 300, 250, 0, 250
	approach 2	7	184	1, 0.976 7, 0.991 8, 0.961 8, 1.00 5, 0.992 7	11.4, 300, 300, 250, 0, 250

6. Conclusions

The main works of this paper are described as follows: 1) Based on the node-branch incidence matrix, a power flow calculation algorithm for the radial distribution network has been derived. Then a three-sequence components decoupled power flow method was proposed by considering the weakly coupling features of the three-phase unsymmetrical distribution network. 2) In order to deal with the common PV nodes, a new approach has been studied. The new method assumes that the positive-sequence voltage magnitude of PV node is sustained at a fixed value, and then the formula to calculate reactive power increment has been derived based on Thevenin equivalent circuit theory.

The tests verified the validity and universality of the algorithm. The results show that the proposed algorithm has good convergence, the calculation process is more simple and the calculation time is significantly reduced. Although there exist errors, the error is relatively small and the system accuracy requirements can be satisfied.

Acknowledgements

The research work was supported by National Key Basic Research Program of China （2013CB228203） and the Scientific Research Fund of Sichuan Provincial Education Department (15ZA0058）.

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Appendixes: The data of the modified 69Bus system

A1. The data of the modified 69Bus system

Branch No	Starting Bus	Ending Bus	Branch resistance (ohm)	Branch reactance (ohm)	A_phase Active Power (kW)	A_phase reactive Power (kvar)	B_phase Active Power (kW)	B_phase reactive Power (kvar)	C_phase Active Power (kW)	C_phase reactive Power (kvar)
1	0	1	0.0005	0.0012	0	0	0	0	0	0
2	1	2	0.0005	0.0012	0	0	0	0	0	0
3	2	201	0	0	0	0	0	0	0	0
4	201	3	0.0015	0.0036	0	0	0	0	0	0
5	3	4	0.0251	0.0294	0	0	0	0	0	0
6	4	5	0.366	0.1864	1.6	1.2	2.6	1.5	1.2	0.8
7	5	6	0.3811	0.1941	15.4	10	10.6	8.5	20.2	11.8
8	6	7	0.0922	0.047	35	18	27.6	10.5	30.2	12.8
9	7	8	0.0493	0.0251	10	7	17.6	9.5	13.2	6.8
10	8	9	0.819	0.2707	11	5	15	7.5	13	6.8
11	9	10	0.1872	0.0691	35	15	50	21.5	63	36.8
12	10	11	0.7114	0.2351	48	24	50	19.5	33	11.8
13	11	12	1.03	0.34	4	1.5	2	1	3	1.1
14	12	13	1.044	0.345	1	0.5	3	1.2	5	1.9
15	13	14	1.058	0.3496	0	0	0	0	0	0
16	14	15	0.1966	0.065	15.5	10	18	13	12	6
17	15	16	0.3744	0.1238	13	10.5	19	15	28	18
18	16	17	0.0047	0.0016	25	15	12	9	20	14
19	17	18	0.3276	0.1083	0	0	0	0	0	0
20	18	19	0.2106	0.0696	1	0.6	1	0.1	1.5	0.6
21	19	20	0.3416	0.1129	46	21	50	20	63	32
22	20	21	0.014	0.0046	1.8	1.5	2.5	1	1	0.5
23	21	22	0.1591	0.0526	0	0	0	0	0	0
24	22	23	0.3463	0.1145	12	8	10	6	10	6
25	23	24	0.7488	0.2745	0	0	0	0	0	0
26	24	25	0.3089	0.1021	4	2	6	2	4	3
27	25	26	0.1732	0.0572	4	3	5	3	7	4
28	2	27	0.0044	0.0108	10	8.6	8	6	8	6
29	27	28	0.064	0.1565	7	4.6	10	8	9	6
30	28	29	0.3978	0.1315	0	0	0	0	0	0
31	29	30	0.0702	0.0232	0	0	0	0	0	0
32	30	31	0.351	0.116	0	0	0	0	0	0
33	31	32	0.839	0.2816	5	3	5	4	6	4
34	32	33	1.708	0.5646	9.5	5	8	5	6	3
35	33	34	1.474	0.4673	2	1.5	2	1.5	2	1.5
36	201	271	0.0044	0.0108	6	3.55	12	10	8	5
37	271	281	0.064	0.1565	12	7.55	6	4	9	7
38	281	65	0.1053	0.123	0	0	0	0	0	0
39	65	66	0.0304	0.355	8	6	9	7	8	5

40	66	67	0.0018	0.0021	9	7	7	5	7	5
41	67	68	0.7283	0.8509	1.2	1	0.5	0.2	0.8	0.3
42	68	69	0.31	0.3623	0	0	0	0	0	0
43	69	70	0.041	0.0478	2	1.5	2	1.5	2	1.5
44	70	88	0.0092	0.0116	0	0	0	0	0	0
45	88	89	0.1089	0.1373	10	6.3	13	9	18	11
46	89	90	0.0009	0.0012	17.2	12.3	14	10	12	7.5
47	3	35	0.0034	0.0084	0	0	0	0	0	0
48	35	36	0.0851	0.2083	29	16.4	21	13	30	18.5
49	36	37	0.2898	0.7091	100	74.5	150	110	135	88
50	37	38	0.0822	0.2011	148	105	105	75	132	80
51	7	40	0.0928	0.0473	12.5	8.3	21	12	15	7
52	40	41	0.3319	0.1114	1.6	0.7	2	1	2	1
53	8	42	0.174	0.0886	1.85	1.5	1	0.7	1	0.7
54	42	43	0.203	0.1034	6.4	5	10	8	10	6
55	43	44	0.2842	0.1447	8	5.2	8	5.2	8	5.2
56	44	45	0.2813	0.1433	0	0	0	0	0	0
57	45	46	1.59	0.5337	0	0	0	0	0	0
58	46	47	0.7837	0.263	0	0	0	0	0	0
59	47	48	0.3042	0.1006	36	22	50	33	42	25
60	48	49	0.3861	0.1172	0	0	0	0	0	0
61	49	50	0.5075	0.2585	284	180	240	160	340	210
62	50	51	0.0974	0.0496	12	9	7	3	16	7
63	51	52	0.145	0.0738	0	0	0	0	0	0
64	52	53	0.7105	0.3619	71	42	101	65	60	43
65	53	54	1.041	0.5302	23	11	16	10	23	16
66	10	55	0.2012	0.0611	4	2	6	3	10	7
67	55	56	0.0047	0.0014	6	4	5	3	7	4
68	12	57	0.7394	0.2444	9	5	11	8	10	7
69	57	58	0.0047	0.0016	10	6	9	3	13	11

A2. The data of the PV node

PV No	Connection Bus	Connection Line resistance (ohm)	Connection Line reactance (ohm)	Total Active Power of PV (kW)
1	88	0.001	0.002	220
2	46	0.001	0.002	300
3	14	0.001	0.002	250
4	52	0.001	0.002	300
5	34	0.001	0.002	200
6	23	0.001	0.002	250